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$$\binom{k}{k}+\binom{k+1}{k}+\binom{k+2}{k}+\cdots+\binom{n}{k}=\binom{n+1}{k+1};$$

$$1\binom{k}{k}+2\binom{k+1}{k}+3\binom{k+2}{k}+\cdots+(n-k+1)\binom{n}{k}=\binom{n+1}{k+1}+(k+1)\binom{n+1}{k+2};$$

$$1\cdot 2\binom{k}{k}+2\cdot 3\binom{k+1}{k}+3\cdot 4\binom{k+2}{k}+\cdots+(n-k+1)(n-k+2)\binom{n}{k}$$
$$=2\binom{n+1}{k+1}+4k^{[1]}\binom{n+1}{k+2}+k^{[2]}\binom{n+1}{k+3};$$

and

$$1\cdot 2\cdot 3\binom{k}{k}+2\cdot 3\cdot 4\binom{k+1}{k}+3\cdot 4\cdot 5\binom{k+2}{k}+\cdots+(n-k+1)(n-k+2)(n-k+3)\binom{n}{k}$$
$$=6\binom{n+1}{k+1}+18k^{[1]}\binom{n+1}{k+2}+9k^{[2]}\binom{n+1}{k+3}+k^{[3]}\binom{n+1}{k+4}.$$

Excellent solutions were also received from E. B. ESCOTT, L. C. MATHEWSON, ELIJAH SWIFT, S. C. WITHERS, and the PROPOSER.

GEOMETRY.

A Correction.—Professor R. A. Johnson, Cleveland, Ohio, has called our attention to an error in Professor Clawson's solution of Geometry problem 467, page 50, of the February MONTHLY.

In lines four and five, Mr. Clawson "refers to two quadrilaterals, using the word in the sense of quadrangle, as being inversely congruent, the corresponding sides being equal and parallel but arranged in opposite orders." Manifestly, two such quadrilaterals cannot be drawn. Mr. Clawson admits the justice of this criticism and says that what he wished to state was the fact that one of the quadrilaterals would have to be turned through two right angles or "inverted" in order to have it similarly placed with the other. Mr. Clawson also says that what he meant by saying in line 5 that the corresponding sides of the quadrilaterals are arranged in opposite order was that the sides are opposite in direction.

Professor Johnson points out that the entire solution can be made rigorous, by deleting the last five words of line 5 and also the word "inversely" wherever it occurs. EDITORS.

474. Proposed by LAENAS G. WELD, Pullman, Ill.

Upon a fixed and constant base stands a system of co-planar triangles, for each of which the radius of the inscribed circle is to that of the circumscribed circle as 1 : 2. What is the locus of the vertices opposite to the given fixed base?

SOLUTION BY J. W. CLAWSON, Collegeville, Pa.

Denote the length of the base by a, the varying sides by r and t, the base angle opposite the side t by θ, radius of incircle by i, and radius of circumcircle by R.

Now

$$R=t/2\sin\theta \text{ and } 2i=(a+r-t)\tan\theta/2.$$

Hence, if $2i=R$, $t=4(a+r-t)\sin^2\theta/2$, from which we obtain, $t=\dfrac{2(a+r)(1-\cos\theta)}{3-2\cos\theta}$.

Again,

$$t^2=a^2+r^2-2ar\cos\theta.$$

Hence,

$$4(a+r)^2(1-\cos\theta)^2 = (a^2+r^2-2ar\cos\theta)(3-2\cos\theta)^2,$$

or

$$(a^2+r^2)(4\cos\theta-5) + 2ar(4+\cos\theta-8\cos^2\theta+4\cos^3\theta) = 0,$$

or

$$(a-r)^2(4\cos\theta-5) = 2ar(1-\cos\theta)(1-2\cos\theta)^2.$$

Now the left side of this equation is essentially negative or zero; and the right side is essentially positive or zero. Hence the only real points on the locus are the points for which both sides are zero. These are the points $r = a$, $\theta = 0$; $r = a$, $\theta = 60°$; $r = a$, $\theta = 300°$.

Hence, the only triangles in the system are the two equilateral triangles on opposite sides of the given base.

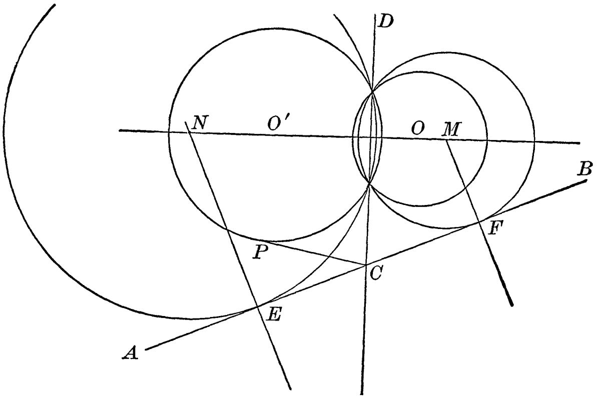
Note.—One other solution, unsigned, was received. This solution was based upon the relation between the radii of the in- and circumcircles and the distance between their centers, namely, $R^2 - d^2 = 2Rr$; since, in the given problem, $r = \frac{1}{2}R$, $d = 0$. Hence, the circles are concentric and the system of triangles reduces to two equilateral triangles on either side of the given line. EDITORS.

475. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Given two circles and a straight line, to draw a circle tangent to the line and coaxial with the two given circles.

SOLUTION BY GEO. W. HARTWELL, Hamline University.

Let O and O' be the two circles and AB the given straight line. Draw the radical axis CD of the two circles. From the point of intersection C of the axis and line AB, construct a tangent to O or O'. Then with C as a center and the length of this tangent, CP, as a radius lay off a distance CE and CF on AB. E and F will be points of tangency. The points, N and M, where perpendiculars erected to AB at E and F meet the line of centers OO' will be the centers of the required circles.



Since the required circle is coaxial with O and O' its center must be on the line of centers OO'. Because the distance along a tangent drawn from any point on the radical axis to each circle of the pencil is constant E or F must be the required point of tangency. Hence, there are two solutions; one circle with center M and radius MF, the other with center N and radius NE.

Also solved by ROGER JOHNSON and H. L. AGARD.

476. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Show that the locus of the middle points of a set of parallel chords intercepted between an hyperbola and its conjugate is $4b^2x^2y^2 - 4a^2y^4 = a^2b^4$.